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An experimental investigation of an axisymmetric jet in a co-flowing air stream

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An experimental investigation of the flow development of an axisymmetric jet exhausting into a moving air stream is made for two values of the ratio of jet velocity to external air velocity. The u-component turbulence intensity and Reynolds shear stress measurements together with the dissipation length scales inferred from measured u-component spectra suggest that the turbulence similarity assumptions are incorrect for the present flow situation. A discussion of the turbulence structure of the flow indicates that self-preservation does not apply for this situation and that the flow far downstream depends strongly on the complete past history.

1. Introduction

When a two-dimensional jet issues into a moving air stream the mean velocity and turbulence structure of the flow gradually change from those associated with the self-preserving pure jet (when $U_0 \gg U_1$, U_0 being the excess velocity at the centre-line over the free-stream velocity U_1), and appear to approach a structure suggesting a self-preserving wake behaviour with $U_0 \ll U_1$. The experimental measurements of Bradbury & Riley (1967) lend some support to this flow picture, while the possible stages of flow development which may occur between the two self-preserving states have been indicated by Bilger (1968) and have been shown to depend on the local value of λ ($\equiv U_0/U_1$), and to some extent on the initial conditions that prevail at the nozzle exit plane. The behaviour of some of the mean properties of this flow such as U_0 and L_0 (the half-width or value of the transverse co-ordinate where the velocity excess over U_1 is equal to $\frac{1}{2}U_0$) has been closely predicted by a method as simple as that of Patel (1971), who uses the momentum integral equation in conjunction with a rather crude auxiliary equation chosen to be compatible with the rate of growth of both the self-preserving jet and wake. The success of this method is surprising when one considers that the strength of the large-scale structure of the wake as measured by, say, the ratio $\overline{uv}_m/U_0^2$ ($\overline{uv}_m$ is the maximum shear stress) is more than twice that in the jet. More sophisticated methods based on the integrated turbulent energy equation and on the assumption of similarity of the structural parameters of the turbulence have been used (Townsend 1966; Bilger 1968; Bradbury 1971) to predict, at least qualitatively, the increase in $\overline{uv}_m/U_0^2$ as the asymptotic wake condition is approached.

In the case of an axisymmetric jet issuing into a co-flowing free stream, the situation appears to be more complex and there is considerable doubt that the turbulence similarity assumption will hold when the flow approaches the asymptotic state of a self-preserving small-deficit axisymmetric wake, if such a state can be realized under laboratory conditions. Recently, Townsend (1970) has indeed shown that, within the framework of the turbulence similarity assumption, no such state appears to be mathematically possible for the axisymmetric wake. Bilger (1969) had earlier arrived indirectly at this conclusion by showing that the turbulent energy equation method failed to predict the axisymmetric jet-wake, the reason for the breakdown being apparently the increased importance of the advection term *vis-à-vis* the production or diffusion terms of the turbulent energy equation. This latter observation is not surprising in view of the possible strong effect of the initial conditions on the establishment of a self-preserving wake. A value of $\overline{uv}_m/U_0^2$ as high as 0·5 was obtained by Carmody (1964) in the wake of a disk. This means that the entrainment constant β (defined by Townsend (1966) as $\beta = [(U_1+\frac{1}{2}U_0)/\frac{1}{2}U_0]\, dL_0/dx$ in the notation of the present paper) could vary by as much as a factor of 20 when one takes into account a value of

$$\overline{uv}_m/U_0^2 = 0{\cdot}017,$$

obtained by Wygnanski & Fiedler (1969) for the self-preserving axisymmetric jet. Clearly, the axisymmetric jet in a moving air stream provides a flow situation in which the turbulence similarity assumption will be severely tested.

One of the aims of the present experimental investigation of an axisymmetric jet in a co-flowing air stream (sometimes referred to as a 'jet-wake' in this paper) is to gain insight into the way the flow structure deviates from that of an axisymmetric self-preserving jet and to establish whether a small momentum excess, self-preserving wake limit is likely. Particular attention is paid in § 3 to the behaviour of $\overline{uv}_m/U_0^2$ and to the turbulence structural parameters such as the ratio of the Reynolds shear stress to the axial turbulence intensity $\overline{u^2}$ and to the behaviour of the dissipation length scale L_ϵ, here defined as $L_\epsilon = (\overline{u^2})^{\frac{3}{2}}/\epsilon$ (ϵ is the viscous rate of turbulent energy dissipation), which is presumably related to a scale representative of the energy-containing eddies. A physical interpretation of the flow structure which is based essentially on the results of § 3 is given in § 4. A brief assessment of existing calculation methods which may be considered for the prediction of the present flow is also given in § 4.

2. Experimental arrangement and techniques

Wind tunnel

A sketch of the jet test-rig is shown in figure 1. The central jet is aligned with the axis of the main tunnel, which has a cross-section 305 by 305 mm with corner fillets. The working section is 183 cm long, has glass side walls and a floor which was adjusted so that the pressure gradient was zero throughout most of the working section. In this series of experiments the jet was supplied from the shop air supply and the volume flow rate determined by measuring the pressure difference across a standard orifice plate mounted in the supply line. The siting of the jet nozzle is such that the boundary layer on the external

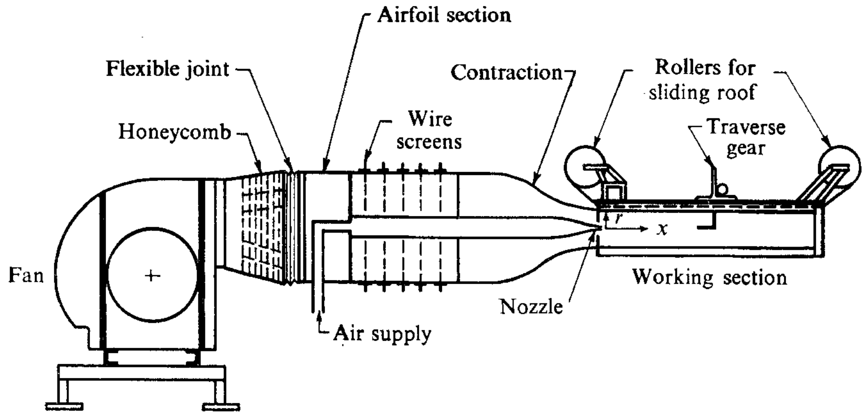

FIGURE 1. Sketch of experimental rig.

contour remains attached. Some measure of control of the external boundary layer can be achieved by means of a suction slot at the beginning of the external contour. Internally the flow contracts from the 50 mm diameter of the supply pipe to the diameter $d = 5{\cdot}28$ mm of the brass nozzle (used throughout this investigation) in a smooth contour, giving a uniform jet exit profile with relatively thin boundary layers. The main tunnel air is supplied by a centrifugal blower, the maximum air speed obtainable being close to 40 m/s. The free-stream turbulence level is 0·1 %.

Vertical and axial traverses of the jet were made through a small slot in a sliding brass-sheet panel in the tunnel working-section roof. Horizontal traverses were also made to establish the precise location of the jet centre and to test for axisymmetry.

Experimental conditions

The free-stream air velocity was kept constant at 30·5 m/s, the static pressure drop across the contraction being continuously monitored on a Betz manometer throughout a run. The two values of the jet velocity U_j used here were 91·3 and 137 m/s. This gave nominal values of U_j/U_1 of 3·0 and 4·5 respectively. The nozzle-exit inside boundary layers were laminar for the above conditions but the external boundary layers were definitely turbulent and had characteristics not too dissimilar from those of a fully developed boundary layer.† The external boundary layers had a thickness of approximately 10 mm with the suction pump operating.

Measuring techniques

Pitot- and static-tube traverses of the flow field were made with pressure differentials recorded with a 'Texas Precision Pressure Gauge'.

A DISA normal hot wire 5 μm in diameter and 1 mm long was operated with a DISA 55 D 01 constant-temperature anemometer. The upper frequency limit of the anemometer was found to be close to 40 kHz (−3 db). Some of the experiments using single normal wires were also recorded on one channel of a Philips Analog-7 tape recorder. This was subsequently played back and digitized

† Mean velocity and turbulence intensity distributions at or near the nozzle exit plane have been measured. The tabulated results are available from the authors.

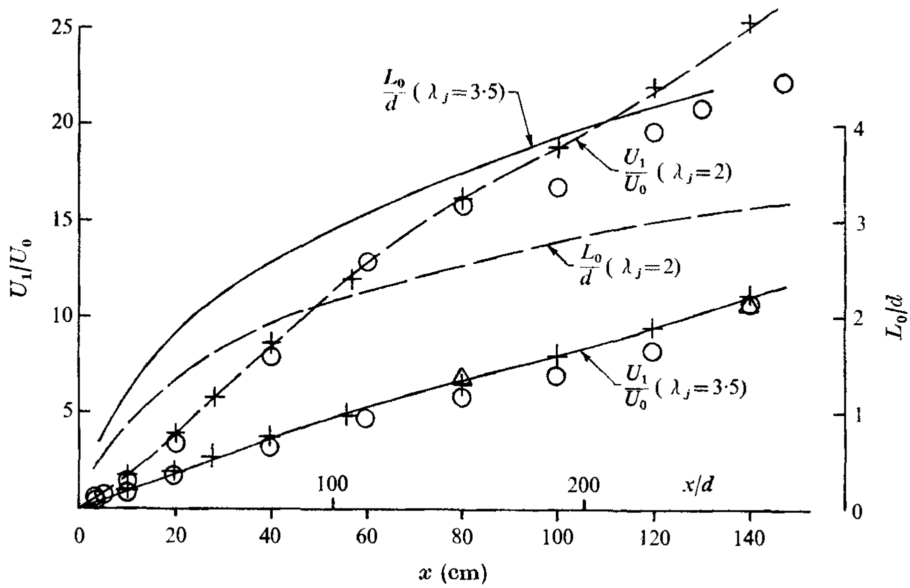

FIGURE 2. Variation of U_0 and L_0.
——, $\lambda_j = 3{\cdot}5$; — —, $\lambda_j = 2{\cdot}0$. ○, hot wire; △, hot wire (radial traverse); +, Pitot tube.

by means of the high-speed digital data acquisition system described in detail in Luxton, Swenson & Chadwick (1967). As the sampling frequency used was equal to 12000 Hz, the signal prior to digitizing was put through a low-pass filter with a sharp cut-off set at 5000 Hz. Whenever possible, the wires were calibrated in both the potential core region of the jet and in the free stream of the test section. The temperature of the laboratory air supply was slightly lower (about 1 °C) than the ambient free-stream air temperature but in the overlapping velocity range the calibrations obtained in the jet and the free stream were identical and no attempt to correct the wire response for possible contamination by this small temperature signal was made. It should be noted, however, that the calibration curves showed a slight deviation from the King's $Re^{\frac{1}{2}}$ law response at speeds in excess of 50 m/s, most likely caused by compressibility effects.

The Reynolds shear stress was obtained by rotating a single inclined hot wire (also 5 μm in diameter and 1 mm long) in the plane of measurement of the jet. The original inclination of the axis of rotation of the wire relative to the axis of symmetry of the jet was determined by viewing through a cathetometer. The effective inclination of the wire was determined by direct yaw calibration in the working-section free stream.

3. Experimental results and discussion

Mean velocity

The mean velocity variation along the centre-line of the jet-wake is shown in figure 2 for the two values of λ_j ($\equiv U_j/U_1 - 1$) investigated here. The centre-line velocity excess U_0 and the half-radius L_0 have been used as the characteristic velocity and length scales respectively to normalize most of the results presented in this section. For $\lambda_j = 3\cdot5$, the distribution of U_1/U_0 is almost linear, as in the case of the axisymmetric jet into still air. For $\lambda_j = 2$, U_1/U_0 increases initially at a faster rate than for $\lambda_j = 3\cdot5$ but at a significantly reduced rate for x/d greater than about 110. The centre-line velocities shown in figure 2 were obtained with both a Pitot tube and with a normal hot wire. The two sets of readings are in reasonable agreement except at the smaller value of λ_j at large values of x/d, where the magnitude of λ ($\equiv U_0/U_1$) is rather low. As the turbulence intensity level relative to U_0 at these low values of λ is reasonably high, the hot-wire results are probably somewhat less reliable than the Pitot-tube values. These latter values have been used to normalize some of the turbulence intensity and shear stress values presented later in this section.

The Pitot mean velocity profiles across the jet-wake cross-section are shown in figure 3 for values of x/d greater than about 38 ($x \simeq 20$ cm). The function $f(\eta)$ is here given by $U = U_1 + U_0 f(\eta)$, where $\eta = r/L_0$. The results in figures 3(*a*) and (*b*) clearly show that the mean velocity profiles become similar at values of x/d as small as 38. The variation with λ of the first three integrals

$$\left(I_n = \int_0^\infty [f(\eta)]^n \eta\, d\eta\right)$$

of the velocity profiles is shown in figure 4 for the two values of λ_j. The constancy of I_n ($I_1 = 0\cdot61$, $I_2 = 0\cdot34$ and $I_3 = 0\cdot23$) at the lower values of λ follows directly from the universality of $f(\eta)$ (figure 3) at the larger x/d values. At the higher λ, I_1 is noticeably smaller, particularly in the case of $\lambda_j = 2$, but the constancy of the parameters I_2 and I_3 is unaffected. The decrease of I_1 is attributed to the persistence of the boundary-layer profile in the outer part of the flow up to values of x/d as high as about 20.

Turbulence intensities

The r.m.s. longitudinal turbulence intensity $(\overline{u^2})^{\frac{1}{2}}$ along the centre-line of the jet-wake is shown in figure 5(*a*). The ratio $(\overline{u^2})^{\frac{1}{2}}/U_0$ rises fairly sharply as the end of the potential core is neared but increases only slightly for $x > 100$ cm, reaching approximately constant values well downstream. In the initial flow region which includes the potential core the turbulence intensity, when normalized by U_j, is much the same for the two values of λ_j (figure 5*b*).

For $\lambda_j \simeq 3\cdot5$ the values of $(\overline{u^2})^{\frac{1}{2}}/U_0$ at $x/d > 75$ are only slightly larger than the value 0·28 obtained by Wygnanski & Fiedler (1969) and various other investigations in the case of a jet into still air. At $\lambda_j = 2$, $(\overline{u^2})^{\frac{1}{2}}/U_0$ appears to asymptote to a value of 0·50, which is lower than the range of values 0·75–1·0 obtained by Cooper & Lutzky (1955) for the flow downstream of either a circular disk or a series of rectangular plates and the value of about 1·0 quoted by Gibson, Chen & Lin (1968) in the wake of a sphere. It should be noted that one might expect this 'asymptotic' value of turbulence intensity to depend on the shape of the wake-generating body, i.e. whether it is blunt or slender, and on the initial conditions such as the nature and thickness of the boundary layers immediately upstream of separation. Carmody (1964) found a value close to

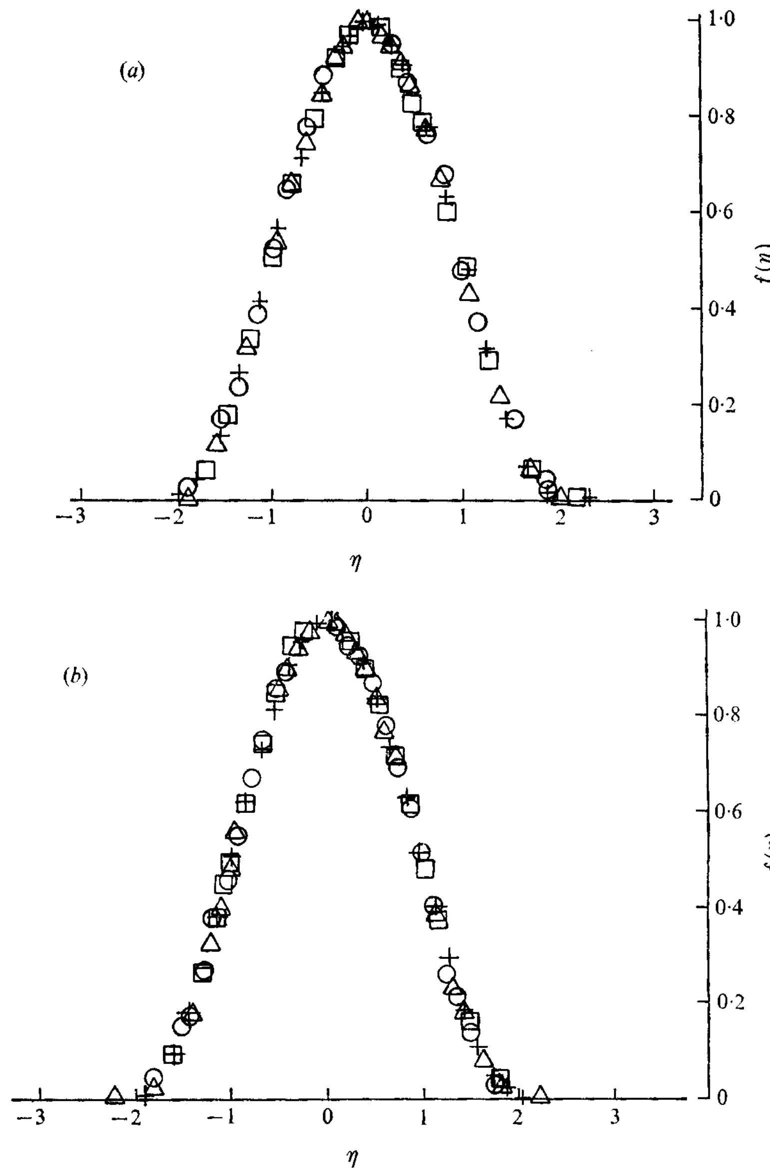

FIGURE 3. Mean velocity distribution. (a) $\lambda_j = 3{\cdot}5$. (b) $\lambda_j = 2{\cdot}0$. x (cm): +, 20; △, 40; □, 80; ○, 140. x/d: +, 38; △, 76; □, 152; ○, 266.

0·70, 15 diameters downstream in the wake of a disk, whilst Chevray (1968) obtained a value as low as 0·27 for the wake of a 6-to-1 spheroid 18 diameters downstream from the stern of the body.

The variation of the u-component intensity across a section of the jet-wake is shown in figures 6 (a) and (b) for various values of x/d. It is interesting to note that for $\lambda_j = 3{\cdot}5$ the distribution of $(\overline{u^2})^{\frac{1}{2}}/U_0$ at $x = 80$ cm is still appreciably different from that at $x = 140$ cm, a result which is supported by the rising trend of $(\overline{u^2})^{\frac{1}{2}}/U_0$ along the centre-line of the flow (figure 5a).

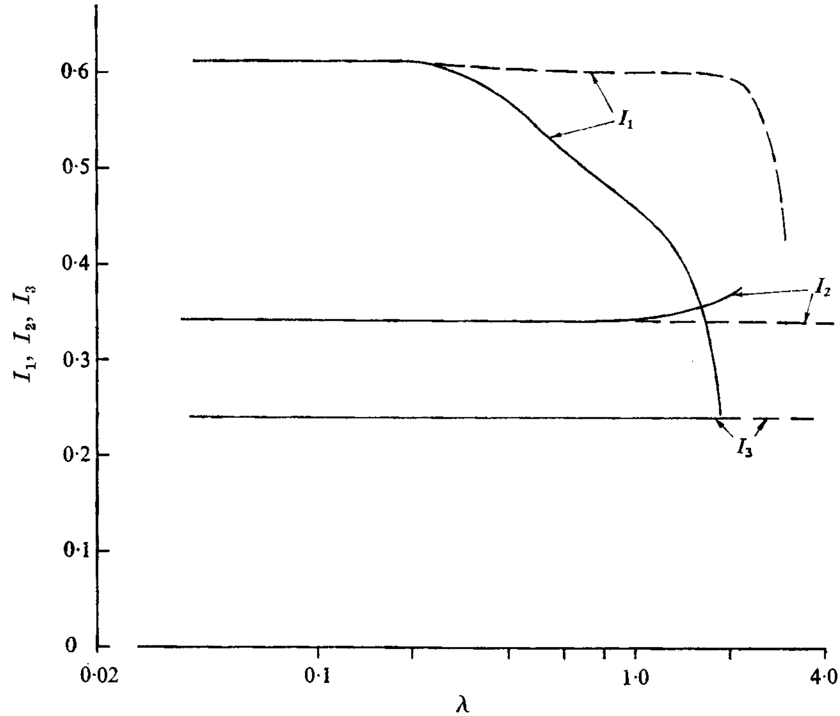

FIGURE 4. Variation of integral parameters I_n with λ. ——, $\lambda_j = 2$; — —, $\lambda_j = 3{\cdot}5$.

Reynolds shear stress

The Reynolds shear stress distributions are shown in figure 7 (*a*) for $\lambda_j = 2$ and in figure 7 (*b*) for $\lambda_j = 3{\cdot}5$. The distributions of figure 7 (*a*) suggest that, within the uncertainty of the measurements, self-preservation may exist for x/d larger than 150. For $\lambda_j = 3{\cdot}5$, the same trend is observable to a somewhat lesser extent, the distribution at $x/d = 248$ remaining slightly higher than the distribution at $x/d = 152$, in agreement with the trend observed in figure 4 (*b*) for the u-intensity distributions. The maximum value of the shear stress occurs near $0{\cdot}7L_0$, which corresponds roughly to the position of the peak in the shear stress results of Wygnanski & Fiedler (1969). The maximum value of $|\overline{uv}|$, taken as the average of the experimental maxima for the positive and negative r sides respectively, has been plotted in figure 8. The results at $\lambda_j = 3{\cdot}5$ again indicate the slowly increasing trend with increasing x/D which was evident in the $(\overline{u^2})^{\frac{1}{2}}/U_0$ plot of figure 5 (*a*). It is interesting to note that these values of $|\overline{uv}|_m/U_0^2$ are significantly higher than the asymptotic value 0·017 found by Wygnanski & Fiedler in the strong jet. The value 0·064 obtained at $x/d = 246$ for $\lambda_j = 2$ is one order of magnitude lower than the (average) value 0·50 obtained by Carmody (1964) for distances greater than about 10 diameters in the axisymmetric wake downstream of a circular disk.† Chevray (1968), however, obtained a value of $|\overline{uv}|_m/U_0^2$

† The results of Carmody (1964) show an amount of scatter rather larger than that for the present and other data available in the literature. It should also be said that the majority of investigations in the axisymmetric wake are confined to a region $x/d < 30$, which is somewhat too small to allow any definitive statement to be made with regard to self-preservation.

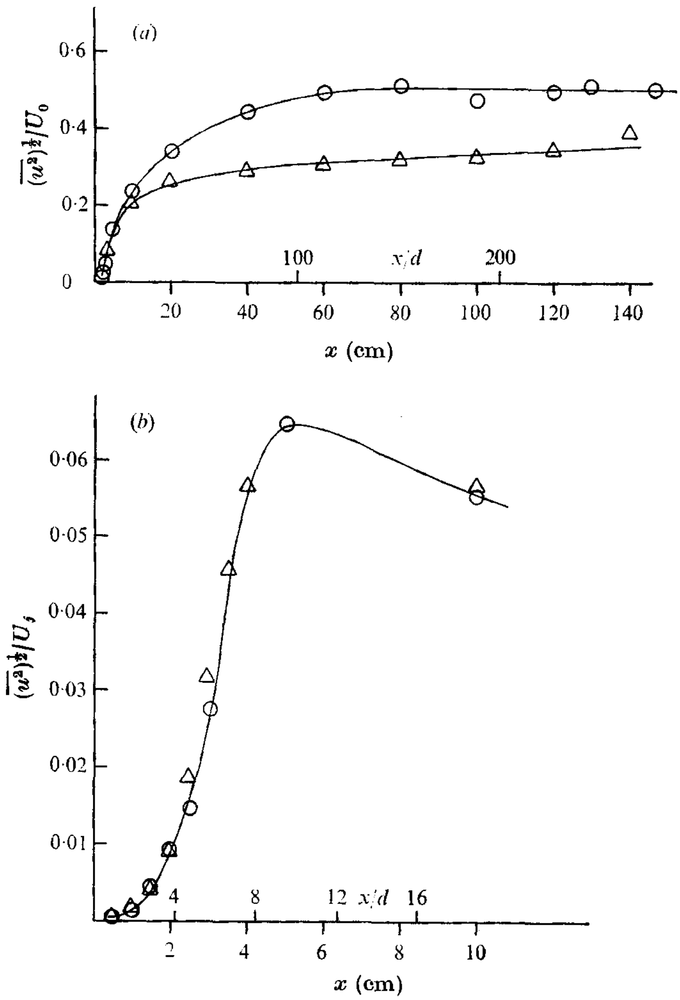

FIGURE 5. u-component turbulence intensity along axis of symmetry. (a) Distribution over the whole flow. (b) Distribution near the nozzle. ○, $\lambda_j = 2$; △, $\lambda_j = 3{\cdot}5$.

as low as 0·035 at 18 (maximum) diameters in the wake downstream of a six-to-one spheroid.

The present results for $|\overline{uv}|_m$ are compared in figure 8 with the values calculated at $r = L_0$ from a knowledge of the mean velocity field. It is assumed that the mean velocity profile is represented by $U = U_1 + U_0 f(\eta)$, where f was shown (figures 3a, b) to remain constant for x greater than about 20 cm but changes appreciably, particularly for $\lambda_j = 2$ (see figure 4), in the region close to the nozzle. The x equation of motion, with the normal stress term neglected, is

$$U\frac{\partial U}{\partial r} + V\frac{\partial U}{\partial r} + \frac{1}{r}\frac{\partial \overline{uv}r}{\partial r} = -\frac{dP}{dx},$$

where the external pressure gradient dP/dx is here negligible for x greater than 40 cm but has been retained in the region immediately downstream of the exit

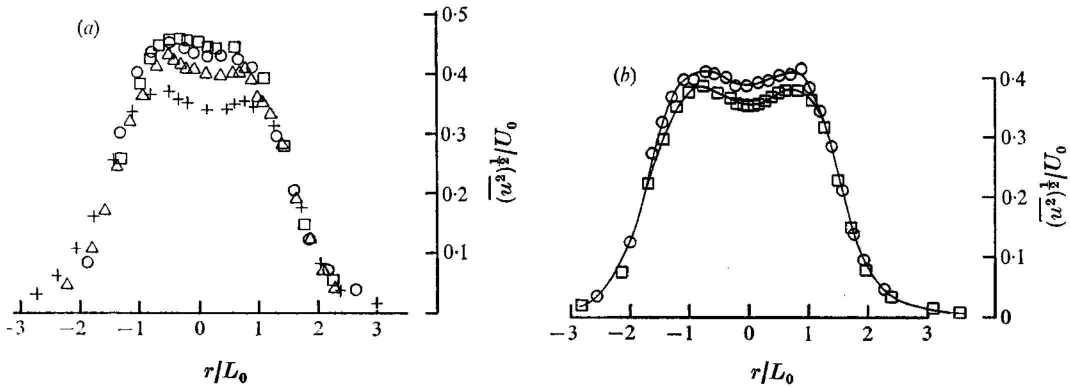

FIGURE 6. u-intensity profiles across the flow.
(a) $\lambda_j = 2$. (b) $\lambda_j = 3{\cdot}5$. x/d: +, 38; △, 76; □, 152; ○, 266.

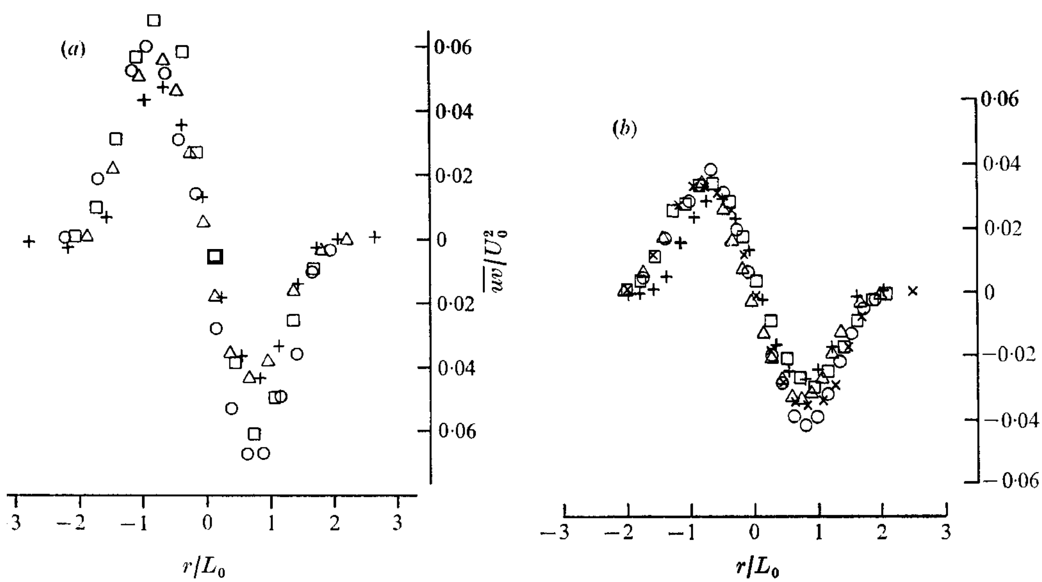

FIGURE 7. Reynolds shear stress profiles.
(a) $\lambda_j = 2$. (b) $\lambda_j = 3{\cdot}5$. x/d: +, 38; △, 96; □, 152; ×, 186; ○, 248.

plane of the nozzle. When integrated between the jet-wake axis and any value r, the above equation leads to

$$\begin{aligned}\overline{uv}r = {} & L_0^2 U_1\left[I_1'\frac{dU_0}{dx}+\frac{U_0}{L_0}I_x'-\frac{U_0}{L_0}\frac{dL_0}{dx}(\eta^2 f-2I_1')\right]\\ & +L_0^2U_0\left[\left(2I_1'-f\frac{\eta^2}{2}\right)\frac{dU_1}{dx}+(2I_2'-fI_1')\frac{dU_0}{dx}+\frac{(2I_{2x}'-fI_x')\,U_0}{L_0}\right.\\ & \left.+2(I_2'-fI_1')\frac{U_0}{L_0}\frac{dL_0}{dx}\right],\end{aligned}$$

where use has been made of the continuity equation

$$\frac{\partial U}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(rV) = 0$$

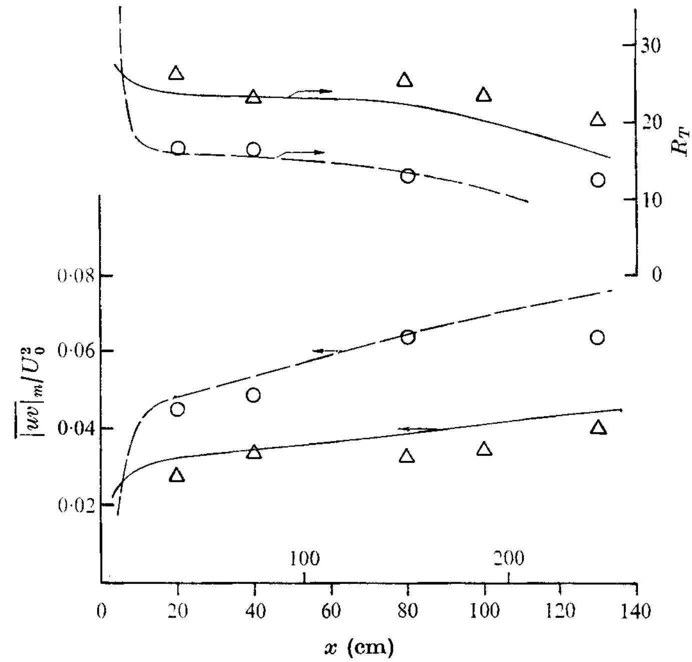

FIGURE 8. Variation of maximum shear stress and eddy viscosity Reynolds number. $\lambda_j = 3{\cdot}5$: ——, calculated; △, measured. $\lambda_j = 2{\cdot}0$: — —, calculated; ○, measured.

and
$$I'_n = \int_0^\eta f^n \eta\, d\eta, \quad I'_x = L_0 \int_0^\eta \frac{\partial f}{\partial x} \eta\, d\eta, \quad I'_{2x} = L_0 \int_0^\eta f \frac{\partial f}{\partial x} \eta\, d\eta.$$

At $\eta = 1, f = \frac{1}{2}, I'_n = J_n, I'_x = L_0(dJ_1/dx)$ and $I'_{2x} = \frac{1}{2}L_0(dJ_2/dx)$, with J_n defined as
$$J_n = \int_0^1 f^n \eta\, d\eta.$$

The above equation, at $\eta = 1$, reduces to
$$\frac{\overline{uv}}{U_0^2} = [(2J_1 - \tfrac{1}{2})\lambda^{-1} + 2J_2 - J_1]\frac{dL_0}{dx} - [J_1 + (2J_2 - \tfrac{1}{2}J_1)\lambda]\, L_0 \frac{d\lambda^{-1}}{dx}$$
$$+ (2J_1 - \tfrac{1}{4})\frac{L_0}{U_0}\frac{dU_1}{dx} + (\lambda^{-1} - \tfrac{1}{2})\, L_0 \frac{dJ_1}{dx} + L_0 \frac{dJ_2}{dx}.$$

The last three terms on the right-hand side of this equation are small compared with the first two, the pressure-gradient term being of the same order as the terms involving the change in the shape of the velocity profiles at least over the initial region of development of the jet-wake. In view of the inaccuracies involved in the above calculation procedure, the agreement shown with the experimental results (for $\eta = 0{\cdot}8$) must be regarded as satisfactory and a useful check on the reduction of the inclined-wire data.

The eddy viscosity Reynolds number $R_T = U_0 L_0/\nu_T$, where
$$\nu_T = -\overline{uv}/(\partial U/\partial y),$$
is determined at $\eta = 1$ by
$$R_T = \left[\frac{\partial f/\partial \eta}{\overline{uv}/U_0^2}\right]_{\eta=1}.$$

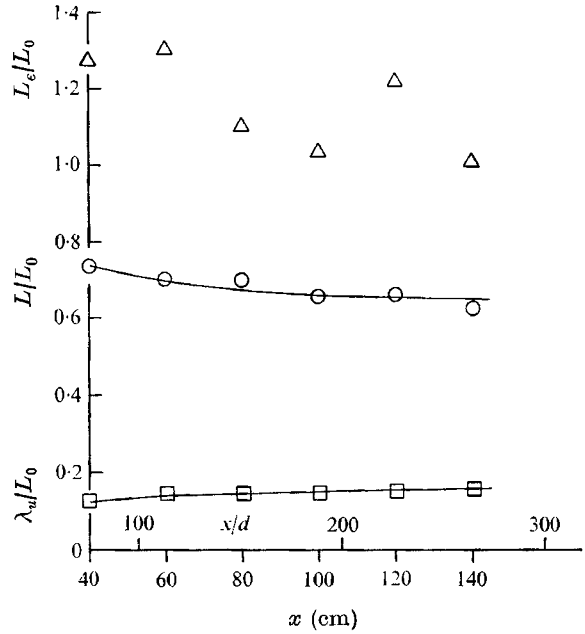

FIGURE 9. Variation of length scales for $\lambda_j = 2$. △, L_ϵ/L_0; ○, L/L_0; □, λ_u/L_0.

The distributions of figure 8 show that after an initial decrease R_T decreases only slowly with increasing x, the minimum value obtained being equal to 12 in the case of $\lambda_j = 2$. This value is significantly higher than the values in the range 1–4 deduced from the data of Cooper & Lutzky (1955), Carmody (1964) and Hwang & Baldwin (1966) for the wake of a circular disk but remains comparable with a value of about 13 for the data of Chevray (1968).

Length scales

The integral length scale L shown in figure 9 for $\lambda_j = 2$ was inferred from the autocorrelation curve of the u fluctuations on the centre-line by making use of the Taylor hypothesis. The autocorrelation curve was obtained indirectly (for the sake of computational speed) by first doing a Fourier transformation of the digital u signal using an FFT algorithm developed by Fraser (1970) and then carrying out an inverse transform of the spectrum. The convection velocity measurements of Wygnanski & Fiedler (1969) showed that the Taylor hypothesis, although reasonable near the centre-line, was rather inadequate in the outer region of the jet. For the present results the condition $\overline{u^2}/U^2 \ll 1$ was closely satisfied both at the centre-line and almost everywhere else in the flow. The value $L/L_0 = 0{\cdot}65$ at the last measuring station is higher than the value 0·45 obtained by Wygnanski & Fiedler (1969) but lower than the value 0·82 obtained by Corrsin & Uberoi (1951). It should be noted that there is a large amount of scatter (almost $\pm 100\%$ relative to the Wygnanski & Fiedler value) in the data for L/L_0 reported in the literature. Wygnanski & Fiedler claim that part of this discrepancy is probably due to the poor low-frequency response of the anemometry and auxiliary electronics, the majority of the investigators having used

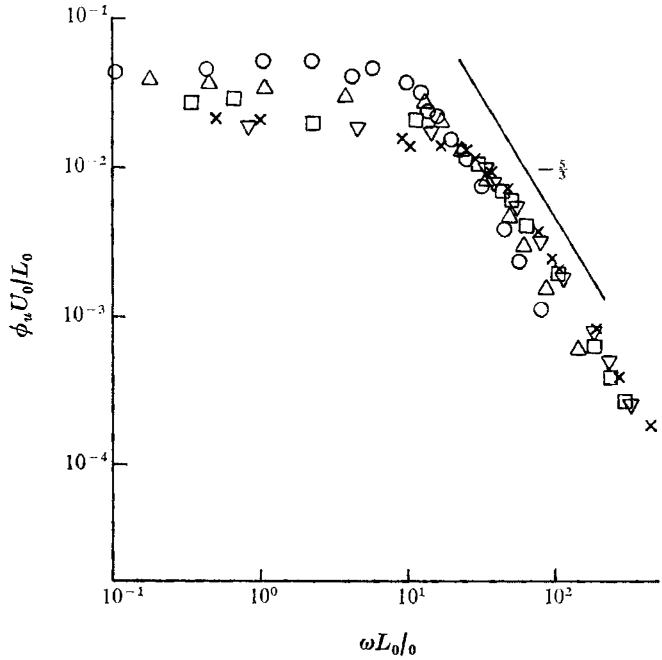

FIGURE 10. u-component spectra. x (cm): ○, 40; △, 60; □, 100; △, 120; ×, 140.

the extrapolation to zero frequency of their low wavenumber part of their one-dimensional u spectrum. The problem of inadequate low-frequency response does not arise here as the one-dimensional ϕ_u spectra of figure 10 are reasonably flat over their low wavenumber range. The normalized spectrum ϕ_u is equal to $\overline{u^2}(\omega)/\overline{u^2}$ and closely satisfies the requirement

$$\int_0^\infty \phi_u \, d\omega = 1.$$

The decrease in ϕ_u at the low-frequency end with increasing x is consistent with the slight decrease of L/L_0 in figure 9 as $L = \frac{1}{2}U\pi[\phi_u]_{\omega=0}$.

The dissipation length scale L_ϵ, which is presumably related to the scale of the energy-containing eddies of the motion and therefore related to L, is usually defined as $L_\epsilon = \overline{uv}^{\frac{3}{2}}/\epsilon$ (ϵ represents the viscous dissipation rate of turbulent energy) in the case of a boundary layer and $L_\epsilon = \overline{q^2}^{\frac{3}{2}}/\epsilon$ in the case of a symmetrical free shear flow. In this latter definition of L_ϵ the value of the total turbulence intensity $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ is usually computed on the plane or axis of symmetry. Here, the normal components $\overline{v^2}$ and $\overline{w^2}$ were not measured and L_ϵ is, for convenience, assumed to be equal to $(\overline{u^2})^{\frac{3}{2}}/\epsilon$. The difficulty of obtaining the dissipation rate ϵ accurately can quickly be ascertained by reference to the literature (see, for example, Lawn 1971). Here, it was decided that perhaps the most reliable means of inferring ϵ was through the use of the relation $\phi(k) = \alpha\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}}$ in the inertial subrange part of the spectrum, k being the one-dimensional wavenumber ω/U. Bradshaw (1967) suggested that within $\pm 15\%$ accuracy α is equal to 0·50, in a region of the flow where the turbulence Reynolds number

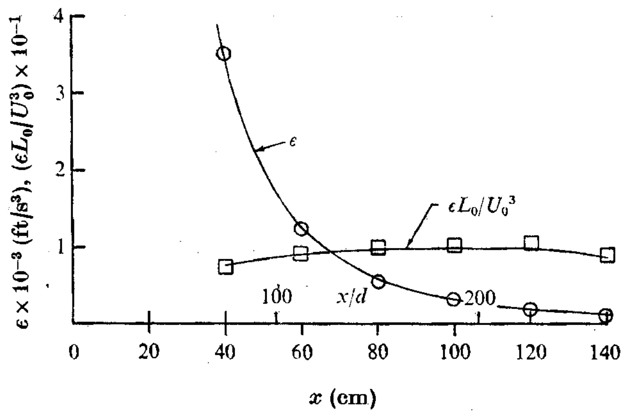

FIGURE 11. Variation of dissipation rate ϵ along the axis of symmetry. $\bigcirc$, ϵ; $\square$, $\epsilon L_0/U_0^3$.

$R_\lambda = (\overline{u^2})^{\frac{1}{2}}\lambda_u/\nu$ (where λ_u is the microscale defined later in this section) is at least greater than 100. In the present flow the spectra of figure 10 suggest that the '$\frac{5}{3}$-region' is present over a fair range of $\omega L_0/U_0$. The present value of R_λ is greater than 100 over the complete range of x considered. Further, the conditions $k \gg 1/L$ and $k \ll 1/\eta$ (η is the Kolmogoroff length scale $(\nu^3/\epsilon)^{\frac{1}{4}}$) are reasonably satisfied in the inertial subrange. It should be noted that the Kolmogoroff frequency is here estimated to lie in the range 30–40 kHz but the sampling frequency was set at the rather low value of 12 kHz (prior to digitizing, the u-signal was passed through a low-pass filter with a sharp cut-off at 5 kHz) as it was not intended to investigate the properties of the small-scale structure of this flow. Since one expects contributions to the dissipation spectrum to remain significant up to values close to the Kolmogoroff frequency, the value of ϵ estimated through the use of the isotropic relation $\epsilon = 15\nu\overline{(\partial u/\partial x)^2}$ may be in serious error (even though there is no real ground seriously to doubt the isotropy of the small-scale motion on the centre-line). The value of ϵ estimated from the '$-\frac{5}{3}$-region' is shown (figure 11) to decrease rather significantly at smaller values of x with the value of the ratio $\epsilon L_0/U_0^3$ remaining surprisingly constant at large x. A necessary condition for the establishment of a 'self-preserving' axisymmetric wake is that $L_0 \sim x^{\frac{1}{3}}$ and $U_0 \sim x^{-\frac{2}{3}}$ thus leading to an expected $\epsilon \sim x^{-2\frac{1}{3}}$ variation (for an axisymmetric self-preserving jet, one expects $\epsilon \sim x^{-4}$). In view of the uncertainty in the value of the 'universal' constant α, the scatter in the L_ϵ/L_0 data of figure 9 must be regarded as fair. The average value of L_ϵ/L_0 indicated by the results for x greater than 80 cm is close to 1·1, which is appreciably below the value 1·82 inferred from the data of Wygnanski & Fiedler.† It should be noted that Wygnanski & Fiedler's measured value of ϵ (by making semi-isotropic assumptions) on the centre-line appears reliable as it is in agreement with the isotropic value $15\nu\overline{(\partial u/\partial x)^2}$ and in agreement with the value inferred by difference from the turbulent energy balance on the jet

† The value 1·82 is not supported by the dissipation value of Gibson (1963) (also inferred from the $-\frac{5}{3}$-relation), which leads to $L_\epsilon/L_0 = 0{\cdot}40$! Gibson's flow appears to have suffered, however, from the rather large length scale of the jet.

Author(s)	Flow	$(\overline{u^2})_0/\lvert\overline{uv}\rvert_m$	L_ϵ/L_0
Wygnanski & Fiedler (1969)	Jet into still air (x/d = 90), $\lambda_j = \infty$	4·6	1·82
Present investigation	Jet into co-flowing stream (x/d = 100)		
	$\lambda_j = 3{\cdot}5$	3·05	—
	$\lambda_j = 2{\cdot}0$	3·90	1·10
Chevray (1968)	Wake of spheroid (x/d = 15), $\lambda_j = 0$	2·33	0·73
Carmody (1964)	Wake of disk (x/d = 20), $\lambda_j = 0$	2·33	0·34

TABLE 1. Turbulence parameters in axisymmetric jets and wakes

axis. The values in table 1 of L_ϵ/L_0 for the measurements of Chevray (1968) and Carmody (1964) are certainly well below those of the present investigation. It should be mentioned that these values of L_ϵ were computed from an averaged dissipation rate across the wake, so that, had the centre-line values of ϵ been used, this would have resulted in even lower values of L_ϵ.

The microscale λ_u was inferred from the values of ϵ via the isotropic relation $\epsilon = 15\nu(\overline{u^2}/\lambda_u^2)$, i.e. with λ_u defined by $\lambda_u^2 = \overline{u^2}/\overline{(\partial u/\partial x)^2}$. The results in figure 9 indicate only a slight increase of λ_u with x. The value 0·16 of λ_u/L_0 at x = 140 cm is four times as large as that given by Wygnanski & Fiedler (1969) at x/d = 90.

4. General discussion

The results presented in the previous section suggest that, although the mean velocity profiles show remarkable similarity at remarkably small values of x/d, the turbulence field does not attain a universal self-preserving state. This introduces serious doubt as to whether the universal self-preserving axisymmetric wake does exist. The present high values of $(\overline{u^2})^{\frac{1}{2}}/U_0$ and $\overline{uv}_m/U_0^2$ for $\lambda_j = 2$ relative to the corresponding values measured in the self-preserving axisymmetric jet together with the breakdown of the turbulence similarity assumption and the significant reduction of the ratio L_ϵ/L_0 make for a rather severe test flow situation for any existing calculation method. It would seem appropriate here to discuss the requirements or applicability of existing calculation methods for predicting the present situation where the flow relaxes from a state pertaining to the axisymmetric jet into still air to one approaching that of a small deficit or excess momentum axisymmetric 'wake'.

Calculation methods such as that of Patel (1971) based on the momentum integral equation and an auxiliary equation are unlikely to work for the present situation as the rather large and non-universal variation of R_T (or the entrainment rate) with λ does not suggest a (simple) universal auxiliary equation. A calculation method which attempts to incorporate some 'history' of the flow is the turbulent energy equation transformation method used successfully by Townsend (1966) and Bilger (1968) in two-dimensional flows. The method essentially transforms the turbulent energy equation into an equation for the entrainment constant β (here defined as $\beta = [(2+\lambda)/|\lambda|]\,dL_y/dx$), when the turbulence similarity assumption is made and when L_ϵ and L_γ, the average position of the jet-wake interface, are assumed to be proportional to L_0.

Bilger (1969) found that no reasonable choice of values for the parameters $\overline{uv}_m/(u^2)_0$, E ($\equiv L_\epsilon/L_0$) and Γ ($\equiv L_\gamma/L_0$) will lead to even an approximate prediction of the axisymmetric jet-wake, complex values of β resulting for values of λ as high as 3. The breakdown of the method suggests that the assumed constancy of the above three parameters, which essentially implies an assumption of a fairly 'strong' structure of the turbulence ('strong' in the sense that invariance of these parameters with the turbulence level, flow geometry or history is implied), needs to be revised for the present flow situation. A static stability analysis of the turbulent energy equation (integrated across the flow) by Bilger (1969) shows that a 'strong' structure is untenable for the axisymmetric wake, where production is less than half the dissipation. In the case of the self-preserving axisymmetric jet, where the advection represents at most about 40% of the production,† the concept of Reynolds number similarity (implying a far-downstream flow independent of the initial or boundary conditions which are basically responsible for the establishment of the turbulence) can hardly be doubted. An example of the fairly rapid obliteration of the effect of initial conditions on the flow of an axisymmetric jet into still air is provided by the recent experiment of Champagne & Wygnanski (1971), where the flow downstream of a double co-axial jet approaches a self-preserving structure fairly rapidly. In the case of the axisymmetric wake, the situation is vastly different. The averaged turbulent energy equation across the flow can be written as

$$\epsilon\pi L_\gamma^2 = -\frac{d}{dx}\int_0^\infty \tfrac{1}{2}U[(U-U_1)^2+\overline{q^2}]\,2\pi r\,dr,$$

where ϵ represents here the mean dissipation of turbulent energy across the flow. With assumptions of similarity for the mean velocity and turbulent energy profiles this equation can be reduced, in the case of a wake, to

$$\epsilon = \frac{U_0^3}{L_0\Gamma^3}\left(I_2+\tfrac{1}{2}k_1\Gamma^2\frac{(\overline{q^2})_0}{U_0^2}\right)\beta,$$

where
$$k_1 \equiv 2\left(\frac{L_0}{L_\gamma}\right)^2\int_0^\infty \frac{\overline{q^2}}{(\overline{q^2})_0}\,\eta\,d\eta$$

(equal to 1 if $\overline{q^2}$ is assumed constant in the turbulent part of the flow). The ratio of advection to production is simply

$$\frac{k_1}{2I_2}\Gamma^2\frac{(\overline{q^2})_0}{U_0^2}.$$

With $I_2 = 0{\cdot}34$, $\Gamma = 1{\cdot}7$ (the available experimental evidence suggests that Γ is not much different for axisymmetric jets and wakes) and $k_1 \simeq 0{\cdot}7$, the present results for $\lambda_j = 2$ indicate that the advection is equal to about twice the production. When the wake data of Chevray (1968) and Carmody (1964) are taken

† These values of production and advection refer to averaged values of $\overline{uv}(\partial U/\partial r)$ and ϵ across the flow.

into account, the advection/production ratio is found to be within the range 1–9. This strongly suggests that the advective transport of turbulent energy and of Reynolds stress in the case of the wake cannot be ignored and that self-preservation, at least for available experimental realizations of the axisymmetric wake, may be suspect.

The data in table 1 show that the assumed constancy of the ratio L_ϵ/L_0 is in urgent need of revision. The relative reduction in L_ϵ as the wake asymptote is approached cannot be reconciled with the increased entrainment rate or the increased standard deviation σ of the mean position of the wake interface (e.g. Demetriades 1968). Whilst one expects that the dissipation length scale L_ϵ ought to be representative of a length scale associated with the energy-containing eddies (at least in a situation where the local energy balance is not grossly violated, i.e. production $\simeq$ dissipation) it seems that the increased importance of advection for the present case undermines this expected scaling. There is little doubt that the strain history of the flow will have a strong effect on the length-scale relationship, and the inclusion of the variation of L_ϵ in any calculation method appears necessary. A method developed by Spalding (1971) which, in addition to the turbulent energy equation, uses a transport equation for the vorticity fluctuations (this is essentially a differential equation for the product of the turbulence energy and L_ϵ) has been found (Kent 1972) to predict reasonably the present flow for $\lambda_j = 2$. The apparent success of this method, which will be discussed in detail elsewhere, underlines the importance of including the variation of L_ϵ in the jet-wake flow development. An algebraic equation for the eddy-viscosity Reynolds number R_T in terms of the mean rate of strain ratio $(\partial U/\partial y)/(\partial V/\partial y)$ has been derived by Gartshore (1964) by first assuming that the large eddy structure is in energy equilibrium (Townsend 1956, p. 127) and then using the mean vorticity equation to relate the mean vorticity of the large eddy to the rate-of-strain ratio. This algebraic equation is effectively an equation for the length scale associated with the large eddy motion. It is unlikely, however, that this equation will be of use in the present flow situation since, as mentioned previously, R_T does not asymptote to a universal value for the axisymmetric wake and what is more important, the length scale relevant to the equation is, unlike L_ϵ, closely proportional to σ, the spread of the interface about its mean position. The full differential equation for L_ϵ used by Spalding (1971) (see also Rotta 1951, 1971) can be deduced from the Navier–Stokes equations but a few non-rigorous assumptions are needed to express some of the correlations in terms of gradients of time-mean quantities. A further improvement on the two differential equations method would presumably be obtained by introducing yet another differential equation which attempts to include the variation of $(\overline{u^2})_0/\overline{uv}_m$ (shown in table 1) along the flow. Consideration of a similar type of equation in two-dimensional flow situations has recently been given by Hanjalić & Launder (1972).

5. Conclusions

The experimental investigation of the flow of an axisymmetric jet in a moving air stream shows that the shape of the mean velocity profiles remains insensitive to the flow development although fairly high levels of u turbulence intensity and Reynolds shear stress are obtained well downstream in the flow. The magnitude of these levels is in keeping with the large entrainment rates expected for the axisymmetric wake but the values recorded are significantly different for the two values of the ratio of jet velocity to external stream velocity used in the experiments. This result together with the variations of L_ϵ/L_0 and $\overline{uv}_m/(\overline{u^2})_0$ indicated in table 1 suggest that the assumption of turbulence similarity, whilst reasonable in the case of a two-dimensional jet-wake, is untenable for the axisymmetric case. The discussion in §4 implies that the role of the large advection component in the turbulent energy budget is probably to weaken considerably the process of obliteration of the effect of initial and boundary conditions in the flow with the result that the turbulence will not achieve an equilibrium structure.

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